MATHEMATICAL MODELLING OF AQUIFER SYSTEMS IN THE SUMÉ VALLEY

BY

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RESUME-- A mathematical modelling for the aquifer systems in the Sumé valley of the homegeneous micro-region of Cariris Velhos is suggested,by way of finite difference techniques.The steady and the un-steady state model calibration is discussed in the light of the two-dimensional differential equation for the flow phenomenon. The short and long term planning and management of the aquifer systems of the Sumé valley have been suggested for proper utilization of the ground water resources of the region.

INTRODUCTION

The Sumé valley lies in the Sukuru River basin which pertains to the polygon of droughts of the north-east Brazil(Fig.1).It represents the homogeneous micro-region of Cariris Velhos(Nº96) of the Paraiba state and is bounded by Pernambuco state in the south and the west,by Serido Paraibano in the north and with the Agreste Borborema in the east.The area under study is about 110ha,extending more lengthwise than its breadth(Fig.2).The area was investigated during the years 1981-83 by a team of researchers(Sarma,1983), in connection with a research project mainly concerned with the salinity problems and related topics,in which the author participated in the analysis of fluctuations of ground water levels and their interpretation with reference to the hydrology,lithology and the groundand surface water contours . The objective of the study was to identify the zones liable to be water-logged,zones which are likely to contribute to the capillary supply of water in the zone of aeration of plants and to help the small and medium agriculturists in knowing the position of ground water table at various times of the year.This was possible by a net-work of observation wells,in all about 37 of them spread over the entire area and monitered over a period of three years.

Hydrology of the Area of Study

The city of Sumé is about 130km from Campina Grande(Lat.7^0-30's of Long.35^0-56'W, and Altitude 500m above mean sea level).The mean annual precipitation on the valley amounts to 8.55×10^8 m^3 and the mean annual runoff is of the order of $0.36 \times 10^8 m^3$,having thus an anomalous hydraulic deficit,because of high temperatures(24°C) and great evapotranspiration losses of the order of 1200mm per year.As per the Thonthwaite standards,the Index of Water Deficit is between -20 and -40, which represents a semi-arid climate.

The valley is with diversified relief,with generally plain to smoothly undulating lands,reaching 600m elevations in the northern

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portion of the valley. Exist also in certain parts strongly undulating to mountainous areas,which have upto 40% or more of declinations,

PROGRAMME OF STUDY

The 110ha of area chosen for study has principally an agricultural infrastructure,with the small and medium agricullturists actively utilizing the land, cultivating such crops as maize, tomato,peas and even rice in certain parts where the salt problems are duly resolved by chemical treatment of terrains. Fig.2 shows the extent of the area chosen for study,although the total area originally surveyed during 1981-83 included another 125ha, thus totalling to a 235ha or more of cultivable area.However, the piezometers were installed only in the 110ha area shown in the figure.

The study involves in identification of productive zones of particular interest.Previous studies made were generally limited to local areas and therefore give only a fragmentary picture of the regional aquifer system.Together with the survey made during the years 1981-83 regards lithology,fluctuations of ground water levels and other details,which resulted in isohypses and isobaths,facilitating knowledge about the direction of flow of water,as also the identification of surface and ground water divides,which are of great interest to the hydro-geologist for studying the water balance in the region of study, during 1983-84, a geo-physical survey was conducted by the author and Dr.Schuster,H.D. of the Department of Geology with Bodenseewerk Geosystem GGA 30,which facilitated in knowing the total depths of aquifers in the valley at various places,revealing informations to the effect that the crystalline rock was a depths varying from 9 to 15.5m.(Fig.3)

The present study originated by the need of having a basic tool for ground water management ,which could be utilised in the regional multi-purpose water resources development and management. The problem posed entails in proceeding with the creation of an observation structure to be utilised for systematic aquifer evaluation and control,both during the period of investigation and for future use and supplying with an hydro-geological synthesis of the aquifer system on the basis of data collected during the course of investigation and from information provided by various organs of public and private interest and constructing a mathematical model which would supply with useful results for the purpose of preliminary planning and which in future,with more details on hand, could be used for better ground water management,within the framework of regional water resources planning.

BASIC DATA NEEDED

In view of the time and cost constraints involved, once that a global frame of reference is defined based on available data, it can be established to carry out only the strictly needed investigations to carryout the basic knowledge of the aquifer system and allow the construction of a preliminary mathematical model. For this purpose, the operations which shall be planned for eventual implimentation would include the following:a definition of the regional observation network, seasonal systematic surveys of piezometric levels and specific conductivity of waters for the purpose of quality studies, pumping tests to determine the specific capacity or storage coefficient and transmissivity coefficient of the aquifer, estimation of temporal and spatial distribution of withdrawl rates of water extracted from aquifers for domestic and agricultural use and estimation of vertical recharge or infiltration using the hydrological balance of agricultural lands.

MATHEMATICAL MODELLING OF AQUIFER

The hydrological scheme of the aquifer system is based on the Equivalent Single Layer Aquifer(Pizzi,1982), the differential flow equation for which is given by

$$\frac{\partial}{\partial X}\left(T\frac{\partial H}{\partial X}\right)+\frac{\partial}{\partial Y}\left(\frac{\partial H}{\partial Y}\right)=Q+S\frac{\partial H}{\partial t} \quad \text{-----} \quad (1)$$

which represents a two-dimentional flow system, in which H = piezometric level in meters,T= transmissivity coefficient,m^2/s, S = storage coefficient,non-dimentional and Q= the sourse term,$m^3/s/m^2$ Although the above equation is valid for confined flow in aquifers the same may be used for phreatic aquifers(free aquifers or un-confined aquifers), the error introduced being negligible due to the relative importance of the aquifer thickness. The solution of this equation can be obtained utilising the finite difference Crank - Nicholson Implicit Procedure(Prickett and Lonnquist,1971).

The study area is divided into definite number of meshes of 50m side and the boundary conditions to be imposed include a) zero flow rate in accordance with impervious outcrops b)imposed potential with Sukuru River and other open drains as boundaries and c) imposed flow rate at outlet of the basin/sub-basin.Fig.2 explains these conditions quite vividly.However,the model implimenatation demands some extra information needed for resolving the problem,such as the source term consisting of the algebraic sum of the withdrawl rates,supposedly to be spread over the meshes of the grid,together with information of the hydrological balance of the agricultural land,which in turn depends on the crop distribution, pedological charactristics of the soils, climatology, quantity and frequency of irrigations delivered to various crops that are grown by the farmers.

STEADY STATE MODEL CALIBRATION

As the observed seasonal fluctuations of the piezometric levels have limited amplitude, the measured three-yearly averages can be therefore assumed to be representative of a pseudo steady state condition of aquifer system which can be utilised as a reference piezometric configuration for the steady state model calibration.

The estimated yearly values of ground water production

and vertical recharge are used to compute the corresponding steady state piezometric distribution. A reasonable fitting of measured and computed piezometric distribution could be reached by a trial and error procedure. It is however to be noted that the transmissivity distribution should show marked correspondence with the known geological and lithological structure of the aquifer system. The yearly volumetric balance of the aquifer system established during calibration could include such items as:Inflow consisting of infiltration in the higher,medium and lower plains, inlet through river fan-heads and seepage of open channels at higher levels and through defective drains.Outflow might include water utilization for domestic and agricultural needs and outflow towards Sukuru river.

UNSTEADY STATE MODEL CALIBRATION

The objective of this study is to evaluate and identify the storage coefficient distribution in the area of study.The period chosen for this purpose could be the hydrological year 1982,during which the initial piezometric configuration was taken as a reference.The monthly ground water withdrawl rates could be obtained by applying the average annual values of the classes of water utilization and the monthly infiltration obtained directly from the hydrological balance of the agricultural lands.

The first guess of the distribution of the storage coefficient could be the one obtained from the pumping test data.The unsteady state response of the aquifer system could be characterised by a seasonal fluctuations of piezometric levels which would be of greater amplitudes than the ones observed with aquifer behaviour under steady state.One other way of arriving at the storage coefficient is by way of computing the same as the product of sand porosity and ratio of thickness of sandy layers to the full thickness of the aquifer(known also as the useful thickness of the aquifer).One of these values would prove to be satisfactory with the observed aquifer behaviour.. Based on piezometric fluctuations, three or four typical zones within the study area could be identified in the free or confined zones.The unsteady state calibration allows monthly balance of the aquifer to be evaluated, which shows balance to be positive in the wet season with maximas during the wet months, and negatives during dry season,with minimas at the beginning and end of irrigation seasons.

AGRICULTURAL MANAGEMENT USING MODELS

One should recognise that the function of ground water resources is irreplaceable and only the correct utilization of these resources would assure feasilibility of the project. The results of the study should therefore confirm the strategic role of the resources and permit better utilization by way of rigorous management of the system within the future regional plaaning activities. The realised model could be utilised for future agricultural management ,with due respect to the limitations involved.

CONCLUSIONS

The development of the model is still in its initial stage, because of certain lacunas found in the data collected on the piezometric head during the years 1981-83. Also, some additional information is being collected about the soil characteristics and the phisiography of the area of study. Once these are ready, the steady state model will be available for use, although for the unsteady state model, the pumping tests are to be yet conducted, and the transmissivity and storage coefficients are to be determined, together with the analysis by Jacob's two-well method with the idea of finding any impervious barriers or sources of recharge within the area of study.

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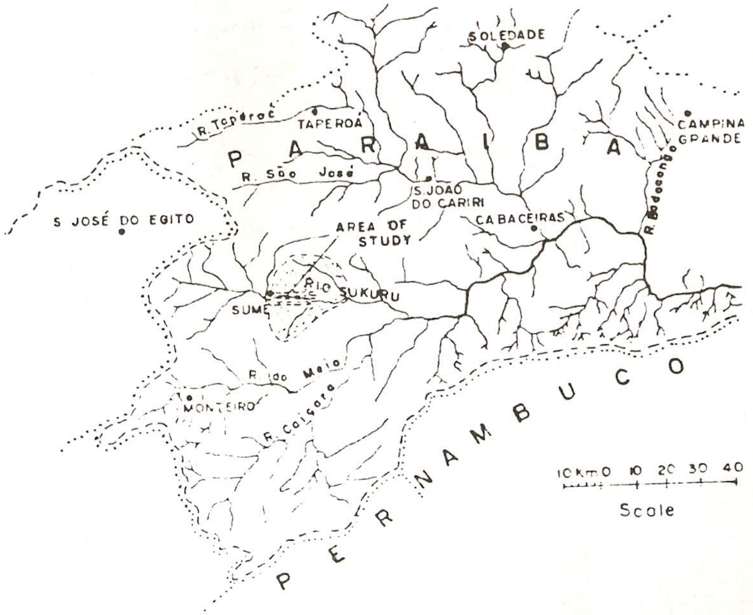
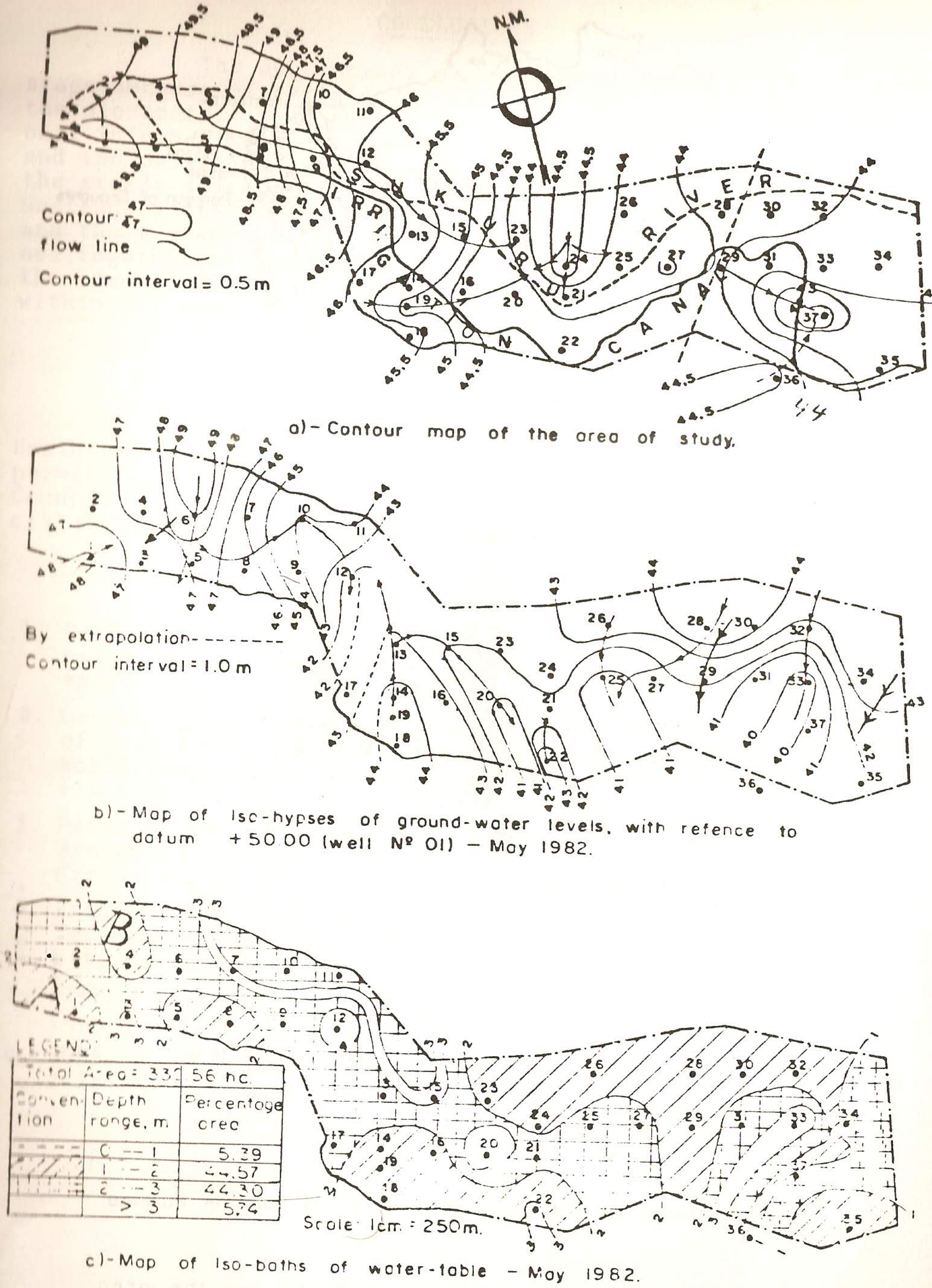
A - Map of Brazil showing the Polygon of Droughts in NE, and Sumé in Paraiba.



B - Detail of the Sukuru River Basin of Sumé in Paraiba.

Fig. 01 - The Polygon of Droughts in Brazil and the area of study in Sumé.



a) - Contour map of the area of study.

b) - Map of Iso-hypses of ground-water levels, with refence to datum + 50.00 (well Nº 01) - May 1982.

Total Area = 331.56 ha.		
Convention	Depth range, m	Percentage area
	0 — 1	5.39
	1 — 2	44.57
	2 — 3	44.30
	> 3	5.74

c) - Map of Iso-baths of water-table - May 1982.

Fig. 02 — Surface contours, Iso-hypses and Iso-baths in the irrigation district of Sumé, Pb.

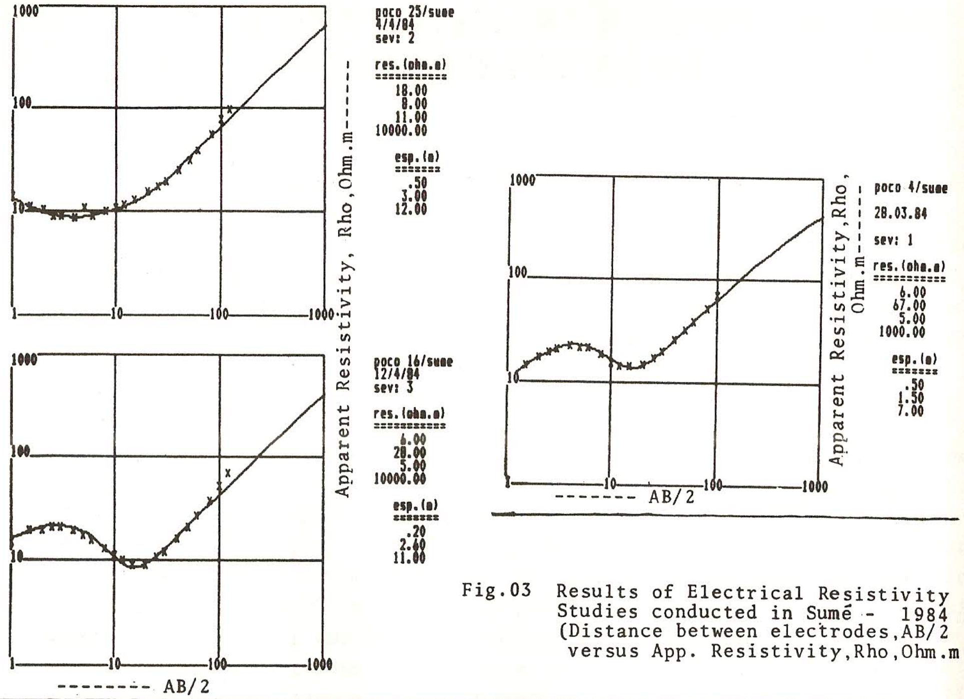


Fig.03 Results of Electrical Resistivity Studies conducted in Sumé - 1984 (Distance between electrodes, AB/2 versus App. Resistivity, Rho, Ohm.m)